

$$\begin{aligned}
\tilde{A}_{\mathfrak{I}} &= U_{\mathfrak{I}}^* A V_{\mathfrak{I}} = \begin{bmatrix} u^* \\ u_{\mathfrak{I}}^* \\ \vdots \\ u_m^* \end{bmatrix} A \begin{bmatrix} v & v_{\mathfrak{I}} & \dots & v_n \end{bmatrix} \\
&= \begin{bmatrix} u^* \\ u_{\mathfrak{I}}^* \\ \vdots \\ u_m^* \end{bmatrix} \begin{bmatrix} Av & Av_{\mathfrak{I}} & \dots & Av_n \end{bmatrix} \\
&= \begin{bmatrix} u^* \\ u_{\mathfrak{I}}^* \\ \vdots \\ u_m^* \end{bmatrix} \begin{bmatrix} \sigma_{\mathfrak{I}} v & Av_{\mathfrak{I}} & \dots & Av_n \end{bmatrix} \\
&= \begin{bmatrix} u^* \sigma_{\mathfrak{I}} u & u^* Av_{\mathfrak{I}} & \dots & u^* Av_n \\ \sigma_{\mathfrak{I}} u_{\mathfrak{I}}^* u & u_{\mathfrak{I}}^* Av_{\mathfrak{I}} & \dots & u_{\mathfrak{I}}^* Av_n \\ \vdots & & & \\ \sigma_{\mathfrak{I}} u_m^* u & u_m^* Av_{\mathfrak{I}} & \dots & u_m^* Av_n \end{bmatrix} \\
&= \begin{bmatrix} \sigma_{\mathfrak{I}} & u^* Av_{\mathfrak{I}} & \dots & u^* Av_n \\ \bullet & & & \\ \vdots & & & \\ \bullet & & & \end{bmatrix}
\end{aligned}$$

$$A_{\mathfrak{I}} \in \mathbb{M}_{m-\mathfrak{I},n-\mathfrak{I}} \, ,$$